DETERMINANTS

Determinant :-

To every matrix $A = [a_{ij}]$ of order n, we can associate a number (Real or complex) called determinant of the square matrix A, where

$$a_{ij} = (i, j)^{th} \text{ element of } A.$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$

Note:-

It is also denoted by $|A|$ or $\det(A)$ or Δ.

Note:-

(i) For matrix A, $|A|$ is read as determinant of A and not modulus of A.

(ii) Only square matrix have determinant.

Determinant of a matrix of order One

Let $A = [a]$ be the matrix of order one, then determinant of A is defined to be equal to a.

Example:-

$$A = [2]$$

Solⁿ:-

$$|A| = 2$$ Ans

Determinant of a matrix of order two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2x2, then the ~~dert~~ determinant of A is defined as.

$$\det(A) = |A| = \Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{11}a_{22} - a_{12}a_{21}$$

Example:-

Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

Soluⁿ:-

$$\Delta = \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$$

$$= (2\times 2) - (-1 \times 4)$$

$$= 4 \div (-4)$$

$$= 4 + 4$$

$$= 8 \text{ Ans}$$

Determinant of a matrix of order 3x3

Determinant of a matrix of order three can be determinant by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row (or a column).

There are six ways of expanding a determinant of order 3 corresponding to each of three rows (R_1, R_2 and R_3) and three columns (C_1, C_2 and C_3) giving the same value as shows below

Consider the determinant of square matrix $A=[a_{ij}]_{3\times3}$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expansion along first Raw (R_1)

$$\therefore |A| = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31})$$
$$+ a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31}$$
$$+ a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

Example:-

Evaluate the determinant

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

Solu^n:-

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

Expanding along C_3

$$\Delta = 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix}$$

$$= 4(-1-12) - 0(1-8) + 0(3+2)$$

$$= 4(-13) - 0 + 0$$

$$= -52$$ Ans

PROPERTIES OF DETERMINANT

Property 1 :-

The value of the determinant remains unchanged if its Rows and columns are interchanged.

Varification :-

Let $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

AND $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$\therefore \Delta = \Delta_1$

Note:- It follows from above property that if A is a square matrix, then

$\det(A) = \det(A')$ where A' is transpose of A.

Property 2 :-

If any two Rows (or columns) of a determinant are interchange, then sign of determinant changes.

verifacation:-

$$\text{Let} \quad \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

If interchanging first and third rows then new determinant obtained is given

$$\Delta_1 = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

$$\boxed{\therefore \quad \Delta = -\Delta_1}$$

Property 3 :-

If any two rows (or columns) of a determinant are identical (all correspoing elements are same) then value of determinant is zero.

Example :-

Evaluate

$$\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$$

Solu^n :-

$$\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$$

Expanding along first row (R_1)

$$\Delta = 3\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix}$$

$$= 3(6-6) - 2(6-9) + 3(4-6)$$

$$= 0 - 2(-3) + 3(-2)$$

$$= 6 - 6 = 0$$

Here R_1 and R_3 are identical.

Property 4 :-

If each element of a row (or column) of a determinant is multiplied by a constant K, then its value gets multiplied by K.

Verification :-

$$\text{Let } \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_1 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix}$$

AND Δ_1 be the determinant obtained by multiplying the elements of the first row by K.

then,

$$\Delta_1 = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Expanding along R_1, we get

$$\begin{aligned} \Delta_1 &= Ka_1(b_2c_3 - b_3c_2) - Kb_1(a_2c_3 - c_2a_3) + Kc_1(a_2b_3 - b_2a_3) \\ &= K[a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3)] \\ &= K\Delta \end{aligned}$$

Hence $\begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = K \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

Note :-

(i) By this property, we can take out ~~any~~ any common factor from any one row or any one column of a given determinant.

(ii) If corresponding elements of any two rows (or columns) of a determinant are proportional (In the same ratio), then its value is zero.

Example :-

Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Solun :-

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \cdot 0 = 0 \text{ (If } R_1 = R_3\text{)}.$$

Property 5 :-

If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants)

Example :-

$$\begin{vmatrix} a_1+d_1 & a_2+d_2 & a_3+d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Property 6 :-

If to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same.

i.e, the value of determinant remain same if we apply the operation

$$R_i \to R_i + K R_j \quad \underline{\text{OR}} \quad C_i \to C_i + K C_j$$

Verification :-

$$\text{Let } \Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\text{and } \Delta_1 = \begin{vmatrix} a_1 + Kc_1 & a_2 + Kc_2 & a_3 + Kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

where Δ_1 is obtained by the operation

$$R_1 \to R_1 + K R_3$$

Here, we have multiplied the elements of the third row (R_3) by a constant K and added them to the corresponding elements of the first row (R_1).

Symbolically :-

We write this operation as

$$R_1 \rightarrow R_1 + KR_3.$$

Now again

$$\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} Kc_1 & Kc_2 & Kc_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + K \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$= \Delta + 0$ (Since R_1 and R_3 are same)

Hence $\boxed{\Delta_1 = \Delta}$

Note :-

(i) If Δ_1 is the determinant obtained by applying

$R_i \to KR_i$ or $C_i \to KC_i$ to

the determinant Δ, then

$$\Delta_1 = K\Delta.$$

(ii) If more than one operation like

$R_i \to R_i + KR_j$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operation.

Example :-

Prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6a+3c \end{vmatrix} = a^3.$$

Solu^n :-

$$L.H.S = \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6a+3c \end{vmatrix}$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$, we have

$$\begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

Now applying

$R_3 \to R_3 - 3R_2$, we have

$$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

Expanding along C_1, we have

$$= a\begin{vmatrix} a & 2a+b \\ 0 & a \end{vmatrix} + 0\begin{vmatrix} a+b & a+b+c \\ 0 & a \end{vmatrix} + 0\begin{vmatrix} a+b & a+b+c \\ a & 2a+b \end{vmatrix}$$

$= a(a^2 - 0) - 0 + 0$

$= a^3 = R.H.S$ proved

AREA OF TRIANGLE

In ~~ealier~~ earlier class, we have studied that the Area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by expression.

Then

$$\text{Area of } \Delta = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right],$$

Now this expression can be written in the form of a determinant as

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Note :—

(i) Since area is a positive quantity we always take the absolute value of the determinant.

(ii) If Area is given, use both positive and negative value of the determinant for calculation.

(ii) The Area of the triangle formed by three collinear points is zero.

Example:–

Find the area of the triangle whose vertices are (3,8), (–4,2) and (5,1).

Soln:–

The are of triangle is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$$

$$= \frac{1}{2}(3+72-14)$$

$$= \frac{61}{2}$$ Ans

MINORS AND COFACTORS

Definition of Minors :-

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row and j^{th} column in which element a_{ij} lies. ~~Mo~~ Minor of an element a_{ij} is denoted by M_{ij}.

Note :-

Minor of an element of a determinant of order $n \, (n \geq 2)$ is a determinant of order $n-1$.

Example :-

Find the minor of element 6 in the determinant

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Solu^n:-

Since 6 lies in the second row and third column, its minor M_{23} is given by

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$$

$$= 8 - 14$$

$$= -6$$

Definition of cofactor :-

cofactor of an element a_{ij}, denoted by A_{ij} is defined by

$$\boxed{A_{ij} = (-1)^{i+j} \cdot M_{ij}}$$, where M_{ij} is minor of a_{ij}

Example :-

Find minor and cofactors of all the element of the determinant

$$\begin{vmatrix} 1 & -3 \\ 5 & 6 \end{vmatrix}$$

Solu^n :-

Minor of the element a_{ij} is M_{ij}.

then

$a_{11} = 1$, so $M_{ij} = M_{11} =$ minor of $a_{11} = 6$

$M_{12} =$ minor of element $a_{12} = 5$

$M_{21} = -3$

$M_{22} = 1$

Now, cofactor of a_{ij} is A_{ij} so,

$A_{11} = (-1)^{1+1} \cdot M_{11} = (-1)^2 \cdot 6 = 6$

$A_{12} = (-1)^{1+2} \cdot M_{12} = (-1)^3 \cdot 5 = -5$

$A_{21} = 3$

$A_{22} = 1$

Note:-

It is denoted by C_{ij} or A_{ij}.

ADJOINT AND INVERSE OF A MATRIX

Adjoint of a Matrix :-

The adjoint of a square matrix $A = [a_{ij}]_{n \times n}$, denoted by Adj$A$ is defined as the transpose of the matrix. $[A_{ij}]_{n \times n}$, where A_{ij} is the cofactor of element a_{ij}.

suppose $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

matrix formed by, cofactors of each element is

$$C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

where $A_{11}, A_{12}, A_{13}, \ldots$ are cofactors of elements $a_{11}, a_{12}, a_{13}, \ldots$ respectively

AND

$$\text{Adj } A = C^T = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Example:-

Find adj A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

Solⁿ:-

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$A_{11} = 4$, $A_{12} = -1$, $A_{21} = -3$, $A_{22} = 2$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

Ans

Theorem 1 :-

If A be any given square matrix of order n, then

$$A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I.$$

where I is the identity matrix.

Proof :-

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

$$\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\therefore A(\operatorname{adj} A) = \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$$= |A| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= |A| I$$

$$\therefore A(\operatorname{adj} A) = (\operatorname{adj} A)A = |A| I.$$

Example:-

i) Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$\therefore$ L.H.S $= A(\text{adj}A)$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4-3(-1) & -6+6 \\ 4-4 & -3+8 \end{bmatrix}$$

$$= \begin{bmatrix} 8-3 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5I$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 8-3 = 5$$

$\therefore$ R.H.S $= |A| I = 5I$ proved

Singular Matrix :-

A square matrix A whose determinant is zero is called a singular matrix.

Example :-

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$\therefore |A| = 8 - 8 = 0$

then the determinant A is zero.

$\therefore$ A is a singular matrix.

Non Singular Matrix :-

A square matrix whose determinant is not zero is called a non-singular matrix.

Example :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \neq 0$$

Therefore A is a non singular matrix.

Theorem 2 :-

If A and B are non singular matrices of the same order, then AB and BA are also non singular matrices of the same order.

Example :-

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 5 & 1 \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+10 & 3+2 \\ 0+20 & 9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 5 \\ 20 & 13 \end{bmatrix}$$ is a non singular matrix.

AND

BA is a also non singular matrix.

Theorem 3:-

The determinant of the product of matrices ~~of~~ is equal to product of their respective determinants, that $|AB| = |A||B|$, where A and B are square matrices of the same order.

Example:-

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5+2 & 3+4 \\ 15+4 & 9+8 \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 19 & 17 \end{bmatrix}$$

$$L.H.S \doteq |AB| = \begin{vmatrix} 7 & 7 \\ 19 & 17 \end{vmatrix}$$

$$= 7\times 17 - 7\times 19$$

$$= 7(-2)$$

$$= -14$$

$$R.H.S \doteq |A|.|B| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} . \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= (4-6)(10-3) = (-2)(7)$$

$$= -14 = R.H.S \text{ proved}$$

Theorem 4:-

A square matrix A is invertible if and only if A is nonsingular matrix.

Proof:- Let A be an invertible matrix of order n & I be ~~be~~ the identity matrix of order n.

There exists a square matrix B of order n

$\therefore AB = BA = I$

We know that

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$

Dividing by $|A|$, we have

$$A\left(\frac{\text{adj}A}{|A|}\right) = \left(\frac{\text{adj}A}{|A|}\right) = \frac{|A|I}{|A|}$$

or, $A\left(\frac{\text{adj}A}{|A|}\right) = \left(\frac{\text{adj}A}{|A|}\right)A = I$

Now, $\left(\frac{\text{adj}A}{|A|}\right)A = I$

Post multiplication of A^{-1}

$\therefore \left(\frac{\text{adj}A}{|A|}\right)A \cdot A^{-1} = IA^{-1}$

or, $\left(\frac{\text{adj} A}{|A|}\right) I = A^{-1}$

$$\boxed{\therefore \; A^{-1} = \frac{\text{adj} A}{|A|}}$$

Inverse of a Matrix :-

Two non singular matrices A and B are called inverse of each other iff $AB = BA = I$

Inverse of matrix A is usually denoted by A^{-1}.

It is also called reciprocal of a matrix.

Then by definition, we get

$$\boxed{\therefore \; AA^{-1} = A^{-1}A = I}$$

Example :- If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that

$A \cdot \text{adj} A = |A| I$, Also find A^{-1}.

Solu^n :-

$|A| = 1(16-9) - 3(4-3) + 3(3-4)$

$= 1 \neq 0$

Now, $A_{11} = 7$, $A_{12} = -1$, $A_{13} = -1$, $A_{21} = -3$, $A_{22} = 1$

$A_{23} = 0$, $A_{31} = -3$, $A_{32} = 0$ and $A_{33} = 1$

$\therefore \text{adj} A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Now, $A(\text{adj}A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| \cdot I$

Also $A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ Ans